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A Contact Process with a Single Inhomogeneous Site

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The one-dimensional basic contact process is a Markov process for which particles give birth on vacant nearest neighbor sites at rate $\lambda > 0$ and particles die at rate one. We introduce a one-dimensional contact process with a single inhomogeneous site: the evolution is as above except that a particle located at the origin does not die. Let λ_c be the critical value of the basic contact process. We show that for $\lambda \neq \lambda_c$ the upper invariant measures of the inhomogeneous contact process and the basic contact process coincide except at a finite number of sites. The behavior at $\lambda = \lambda_c$ is much more interesting: the upper invariant measure of the inhomogeneous contact process concentrates on configurations with infinitely many particles, while it is known that the critical basic contact process dies out. So a single inhomogeneity may provoke a perturbation unbounded in space. As a byproduct of our analysis we prove that the connectivity probabilities of the critical basic contact process are not summable. We also give a biological interpretation of this model.

KEY WORDS: Inhomogeneous contact process; critical point.

1. INTRODUCTION

We consider the one-dimensional contact process ξ_t which is a Markov process with state space $\{0, 1\}^Z$. For any integer x, $\xi_t(x) = 1$ means that the site x is occupied by a particle at time t and $\xi_t(x) = 0$ means that the site x is vacant at time t. The contact process evolves according to the following rules:

(i) If $\xi_t(x) = 1$, then

$$\lim_{s \to 0} \frac{1}{s} P(\xi_{t+s}(x) = 0 \mid \xi_t) = 1$$

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(ii) If $\xi_t(x)=0$, then

$$\lim_{s\to 0}\frac{1}{s}P(\xi_{t+s}(x)=0\mid\xi_t)=\lambda(\xi_t(x-1)+\xi_t(x+1))$$

Here $\lambda>0$ is a parameter. We denote by ξ_t^A the contact process whose initial configuration has one particle on each site of $A\subset Z$ and no particles elsewhere.

The contact process is an attractive process and this implies that if we start the process with all sites in Z occupied by a particle, the law of the process converges to the upper invariant measure μ_λ which is the largest possible invariant measure. For this and other basic facts about the contact process see Liggett[8] and Durrett.[4] Another obvious invariant measure for the contact process is δ_0 the Dirac measure concentrated on the empty configuration. There exists a critical value λ_c such that if $\lambda\leqslant\lambda_c$, then the upper invariant measure $\mu_\lambda=\delta_0$ and if $\lambda>\lambda_c$, $\mu_\lambda\neq\delta_0$ and μ_λ concentrates on configurations with infinitely many particles.[1]

We now introduce a contact process $\bar{\xi}_t$ with a single inhomogeneous site. The birth and death rates follow (i) and (ii) for this process, too, except that at the origin a particle cannot die. So for $\bar{\xi}_t$ the death rate [appearing in (i)] is 1 at all sites but the origin, where it is 0. The process $\bar{\xi}_t$ is still attractive and if we start with all sites occupied by a particle it converges in law to the upper invariant measure that we denote $\bar{\mu}_\lambda$. We will see below that we may construct ξ_t and $\bar{\xi}_t$ simultaneously in such a way that if $\xi_0\leqslant\bar{\xi}_0$ [i.e., $\xi_0(x)\leqslant\bar{\xi}_0(x)$ for all x in Z], then $\xi_t\leqslant\bar{\xi}_t$ at all times $t\geqslant 0$. This implies that for a fixed λ, $\mu_\lambda\leqslant\bar{\mu}_\lambda$ (i.e., $\int f\,d\mu_\lambda\leqslant\int f\,d\bar{\mu}_\lambda$ for every increasing function f defined on the set of configurations). This is known to imply (see Liggett,[8] Theorem 2.4, Chapter II) the existence of a measure ν_λ whose first marginal is μ_λ, whose second marginal is $\bar{\mu}_\lambda$, and such that

$$\nu_\lambda((\xi,\bar{\xi})\colon \xi\leqslant\bar{\xi})=1$$

Theorem 1. (a) If $\lambda\neq\lambda_c$, then the upper invariant measures μ_λ and $\bar{\mu}_\lambda$ of the contact process and of the inhomogeneous contact process coincide except for a finite number of sites. That is,

$$\nu_\lambda\Big((\xi,\bar{\xi})\colon 0\leqslant\sum_{x\in Z}(\bar{\xi}(x)-\xi(x))<\infty\Big)=1$$

(b) If $\lambda=\lambda_c$, then $\bar{\mu}_{\lambda_c}$ concentrates on configurations with infinitely many particles, while it is known that $\mu_{\lambda_c}=\delta_0$.

So Theorem 1 shows that if $\lambda \neq \lambda_c$, then a single inhomogeneity causes a finite perturbation in space. What is more interesting is that if $\lambda = \lambda_c$, then a single inhomogeneity causes a perturbation which is not bounded in space. The proof of part (a) of Theorem 1 is almost immediate. The proof of part (b) is based on a coupling in Cox *et al.*[3] which follows ideas in Galves and Presutti.[5] These methods are heavily one-dimensional and we do not know what to expect for the corresponding problem in higher dimensions.

An easy consequence of Theorem 1(b) is the following result.

Corollary 1. If $\lambda = \lambda_c$, then the connectivity probabilities are not summable. That is,

$$\sum_{x \in Z} P(\exists t \geqslant 0 \colon \xi_t^0(x) = 1) = \infty$$

In particular, if $P(\exists t \geqslant 0\colon\ \xi_t^0(x) = 1)$ behaves like a power law $|x|^{-k}$, the corollary shows that $k \leqslant 1$.

Bezuidenhout and Grimmett[2] have proved that the connectivity probabilities decay exponentially fast if $\lambda < \lambda_c$ in any dimension.

To complete the picture of the contact process with a single inhomogeneous site, we state next a complete convergence theorem.

Theorem 2. For any $\lambda > 0$ and any initial configuration $\bar{\xi}_0$ we have that

$$\bar{\xi}_t \quad \text{converges in law to} \quad P(\sigma = \infty)\,\delta_0 + P(\sigma < \infty)\,\bar{\mu}_\lambda$$

where $\sigma = \inf\{t \geqslant 0\colon\ \bar{\xi}_t(0) = 1\}$.

There has been some interest in the mathematical biology literature in heterogeneous models of the type we introduce here.[6] For instance, the core and satellite hypothesis in metapopulation dynamics predicts that either most species are present in most patches or occupy only a small fraction of the patches.[7] If we think of the origin as the mainland and the other sites as being islands, we see by Theorem 1(a) that the inhomogeneous contact process is consistent with the core and satellite hypothesis: if $\lambda < \lambda_c$, then the upper invariant measure concentrates on finite configurations, while if $\lambda > \lambda_c$, the upper invariant measure concentrates on infinite configurations. Of course, the sharp transition happens for the basic contact process as well and we use the inhomogeneous site only to avoid extinction.

2. PROOFS OF THEOREM 1 AND COROLLARY 1

We begin by recalling Harris' graphical construction of the contact process (for more details see Durrett[4]). We associate each site of Z with three independent Poisson processes, one with rate 1 and the two others with rate λ. Make these Poisson processes independent from site to site. For each x, let $\{T_n^{x,k}: n \geqslant 1\}$, $k = 0, 1, 2$, be the arrival times of these three processes, respectively; the process $\{T_n^{x,0}: n \geqslant 1\}$ has rate 1, the others rate λ. For each x and $n \geqslant 1$ we write a δ mark at the point $(x, T_n^{x,0})$. We draw an arrow from $(x, T_n^{x,1})$ to $(x+1, T_n^{x,1})$. We also draw an arrow from $(x, T_n^{x,2})$ to $(x-1, T_n^{x,2})$. We say that there is a path from (x, s) to (y, t) if there is a sequence of times $s_0 = s < s_1 < s_2 < \cdots < s_n < s_{n+1} = t$ and spatial locations $x_0 = x$, $x_1, \ldots, x_n = y$ so that for $i = 1, 2, \ldots, n$ there is an arrow from x_{i-1} to x_i at time s_i and the vertical segments $\{x_i\} \times (s_i, s_{i+1})$ for $i = 0, 1, \ldots, n$ do not contain any δ. We use the notation $\{(x, s) \to (y, t)\}$ to denote the event that there is path from (x, s) to (y, t). To construct the contact process ξ_t from the initial configuration A (i.e., there is one particle at each site of A) we let $\xi_t^A(y) = 1$ if there is a path from $(x, 0)$ to (y, t) for some x in A. We use the same Poisson marks to construct $\bar{\xi}_t$ except that for this process we ignore the δ marks located on $\{0\} \times [0, \infty)$. Using this graphical construction, it is clear that for a fixed $\lambda > 0$, ξ_t and $\bar{\xi}_t$ are coupled in such a way that if $\xi_0 \leqslant \bar{\xi}_0$, then at all times t we have $\xi_t \leqslant \bar{\xi}_t$. We need to state a few more properties before turning to the proofs. We define the survival time of a process starting with a single particle located at x by

$$\tau^x = \inf\left\{t > 0: \sum_{y \in Z} \xi_t^x(y) = 0\right\}$$

$$\bar{\tau}^x = \inf\left\{t > 0: \sum_{y \in Z} \bar{\xi}_t^x(y) = 0\right\}$$

We also define the densities of μ_λ and $\bar{\mu}_\lambda$ by

$$\rho_\lambda(x) = \mu_\lambda(\xi \in \{0, 1\}^Z: \xi(x) = 1)$$

$$\bar{\rho}_\lambda(x) = \bar{\mu}_\lambda(\xi \in \{0, 1\}^Z: \xi(z) = 1)$$

Since the transition rates of ξ_t are translation invariant and μ_λ is obtained starting from a translation-invariant initial configuration, μ_λ is translation invariant and $\rho_\lambda(x)$ is a constant that we denote by ρ_λ.

The contact process is said to be self-dual in the sense that for any subsets A and B included in Z, we have

$$P\left(\sum_{x\in B}\xi_t^A(x)>0\right)=P\left(\sum_{x\in A}\xi_t^B(x)>0\right)$$

The inhomogeneous contact process is also self-dual. For more details see Durrett.[4]

By self-duality of the contact process we have

$$P(\bar{\xi}_t^Z(x)=1)=P(\bar{\tau}^x>t)$$

and letting t go to infinity, we get

$$\bar{\rho}_\lambda(x)=\bar{\mu}_\lambda(\xi\in\{0,1\}^Z\colon \xi(x)=1)=P(\bar{\tau}^x=\infty)$$

We can now turn to the proof of Theorem 1.

Proof of Theorem 1(a). Define

$$\sigma^x=\inf\{t\geqslant 0\colon \xi_t^x(0)=1\}$$

We have

$$\bar{\rho}_\lambda(x)=P(\sigma^x<\infty;\bar{\tau}^x=\infty)+P(\sigma^x=\infty;\bar{\tau}^x=\infty)$$

Now observe that if $\sigma^x=\infty$, then $\xi_t^x=\bar{\xi}_t^x$ for all t and since $P(\tau^x=\infty)=0$ if $\lambda\leqslant\lambda_c$, we get

$$\bar{\rho}_\lambda(x)=P(\sigma^x<\infty;\bar{\tau}^x=\infty)\qquad\text{for}\quad \lambda\leqslant\lambda_c \tag{2.1}$$

Pick $a>0$ such that $a\lambda<1$ and observe that the rightmost and leftmost particles of a contact process are dominated by Poisson processes with rate λ. Therefore there are C and $\gamma>0$ such that for all x in Z

$$P(\sigma^x<a\,|x|)\leqslant P(\exists s\leqslant a\,|x|\colon \xi_s^x(0)=1)\leqslant Ce^{-\gamma|x|}$$

We now take care of times larger than $a\,|x|$,

$$P(\infty>\sigma^x>a\,|x|)\leqslant P(\tau^x>a\,|x|)$$

If $\lambda<\lambda_c$, then there are C_1 and γ_1 strictly positive such that for all x

$$P(\tau^x>a\,|x|)=P(\tau^0>a\,|x|)\leqslant C_1e^{-\gamma_1|x|}$$

So for all $\lambda < \lambda_c$ we have C_2 and γ_2 such that for all x

$$\bar{\rho}_\lambda(x) \leqslant C_2 e^{-\gamma_2 |x|}$$

Since the density ρ_λ is zero for $\lambda < \lambda_c$, we have

$$\bar{\rho}_\lambda(x) = \bar{\rho}_\lambda(x) - \rho_\lambda(x) = \nu_\lambda((\xi, \bar{\xi})\colon \bar{\xi}(x) = 1; \xi(x) = 0) \leqslant C_2 e^{-\gamma_2 |x|}$$

By the Borel–Cantelli Lemma and the preceding estimate we get Theorem 1(a) for $\lambda < \lambda_c$.

We now turn to the supercritical case. Fix $\lambda > \lambda_c$; we have

$$0 \leqslant \bar{\rho}_\lambda(x) - \rho_\lambda = P(\bar{\tau}^x = \infty; \tau^x < \infty)$$

where the inequality comes from the fact that the inhomogeneous contact process dominates the contact process and the equality comes from self-duality. We have

$$0 \leqslant \bar{\rho}_\lambda(x) - \rho_\lambda = P(\sigma^x < \infty; \bar{\tau}^x = \infty; \tau^x < \infty) + P(\sigma^x = \infty; \bar{\tau}^x = \infty; \tau^x < \infty)$$

But if $\sigma^x = \infty$, then $\xi_t^x = \bar{\xi}_t^x$ for all $t \geqslant 0$ and therefore the second term on the r.h.s. must be zero. So

$$0 \leqslant \bar{\rho}_\lambda(x) - \rho_\lambda = P(\sigma^x < \infty; \bar{\tau}^x = \infty; \tau^x < \infty)$$

Pick $a > 0$ again such that $a\lambda < 1$ and as explained above we get

$$P(\sigma^x < a\,|x|) \leqslant P(\exists s \leqslant a\,|x|\colon \xi_s^x(0) = 1) \leqslant Ce^{-\gamma |x|}$$

On the other hand,

$$P(\infty > \sigma^x > a\,|x|; \bar{\tau}^x = \infty; \tau^x < \infty) \leqslant P(a\,|x| < \tau^x < \infty) \leqslant C_3 e^{-\gamma_3 |x|}$$

where the exponential decay was proved by Durrett and Griffeath (see Liggett[8]) for $\lambda > \lambda_c$. So we get that there are constants C_4 and γ_4 such that for all x

$$0 \leqslant \bar{\rho}_\lambda(x) - \rho_\lambda \leqslant C_4 e^{-\gamma_4 |x|}$$

Here again the Borel–Cantelli Lemma completes the proof of Theorem 1(a) when $\lambda > \lambda_c$. ∎

Proof of Theorem 1(b). In this proof we fix $\lambda = \lambda_c$ and we introduce a renewal process that we will compare to the inhomogeneous contact process. We define $\xi_t^{0,s}$ to be the contact process starting at time $s \leqslant t$ with a single particle located at the origin. To construct this process we use the same Poisson marks as for ξ_t starting at time s. We define $T_0=0$ and for $k\geqslant 1$

$$T_k=\inf\left\{t>T_{k-1}:\sum_{x\in Z}\xi_t^{0,T_{k-1}}(x)=0\right\}$$

Let the number of renewals up to time t be

$$N(t)=\sum_{i=1}^{\infty}1_{\{T_i\leqslant t\}}$$

We construct the process $\xi_t^{0,T_{N(t)}}$ using the same graphical construction as for the contact process. At time 0 we start $\xi_t^{0,T_{N(t)}}$ with a single particle located at the origin and each time this process dies out (which happens with probability one) we restart it putting a particle at the origin. The crucial observation in what follows is that if the initial configuration of the inhomogeneous contact process $\bar{\xi}_0$ is such that $\bar{\xi}_0(0)=1$, then we have the following coupling at all times t:

$$\xi_t^{0,T_{N(t)}}\leqslant\bar{\xi}_t \tag{2.2}$$

Observe that the sequence $(T_i-T_{i-1})_{i\geqslant 1}$ is i.i.d. and the distribution of T_1 is the same as that of τ^0. Moreover, Durrett[4] has proved that if $\lambda=\lambda_c$, then

$$\lim_{t\to\infty}\sqrt{t}\,P(\tau^0>t)=\infty$$

In particular, τ^0 has infinite expectation. Cox *et al.*[3] used this to prove the following result.

Lemma 2.1. For any integer L there exists a sequence of integers j_n going to infinity such that

$$\lim_{n\to\infty}P\left(\sum_{x\in Z}\xi_{j_n}^{0,T_{N(j_n)}}(x)\leqslant L\right)=0$$

For a proof see Lemma 2 in ref. 3. We have

$$\begin{aligned}\bar{\mu}_{\lambda_c}\left(\bar{\xi}:\sum_{x\in Z}\bar{\xi}(x)\leqslant L\right)&=\int P\left(\sum_{x\in Z}\bar{\xi}_{j_n}(x)\leqslant L\right)d\bar{\mu}_{\lambda_c}(\bar{\xi}_0)\\&\leqslant P\left(\sum_{x\in Z}\xi_{j_n}^{0,T_{N(j_n)}}(x)\leqslant L\right)\end{aligned} \tag{2.3}$$

where the equality comes from the stationarity of $\bar{\mu}_{\lambda_c}$ and the inequality comes from the coupling (2.2). We may use this coupling since it is clear that the upper invariant measure of the inhomogeneous contact process has the property

$$\bar{\mu}_{\lambda_c}(\bar{\xi}: \bar{\xi}(0) = 1) = 1$$

Letting first j_n go to infinity and using Lemma 2.1 and then letting L go to infinity in (2.3), we get

$$\bar{\mu}_{\lambda_c}\left(\bar{\xi}: \sum_{x \in Z} \bar{\xi}(x) < \infty\right) = 0$$

and this proves Thorem 1(b). ∎

Proof of Corollary 1. By (2.1) we have that

$$\bar{\rho}_{\lambda_c}(x) \leqslant P(\sigma^x < \infty)$$

If the series $\sum_{x \in Z} P(\sigma^z < \infty)$ were finite, so would be the series $\sum_{x \in Z} \bar{\rho}_{\lambda_c}(x)$. But this would imply by Borel–Cantelli that $\bar{\mu}_{\lambda_c}$ concentrates on configurations with a finite number of particles, contradicting Theorem 1(b). Therefore

$$\sum_{x \in Z} P(\sigma^x < \infty) = \infty \qquad \text{for} \quad \lambda = \lambda_c$$

This proves Corollary 1. ∎

3. PROOF OF THEOREM 2

The case $\lambda < \lambda_c$ is easy to deal with. We have two recurrent classes for $\bar{\xi}_t$: one consisting of the empty state and the other

$$\left\{\bar{\xi}: \sum_{x \in Z} \bar{\xi}(x) < \infty; \bar{\xi}(0) = 1\right\}$$

The upper invariant measure concentrates on the preceding class, which is countable. By elementary properties of Markov chains on countable spaces there is a unique invariant measure concentrating on each recurrent class. The complete convergence theorem follows for $\lambda < \lambda_c$.

We now turn to the case $\lambda \geqslant \lambda_c$. We start by showing that the convergence in Lemma 2.1 holds more generally, that is,

$$\lim_{t \to \infty} P\left(\sum_{x \in Z} \xi_t^{0, T_{N(t)}}(x) \leqslant L\right) = 0 \qquad \text{for all} \quad \lambda \geqslant \lambda_c \tag{3.1}$$

We have

$$P\left(\sum_{x \in Z} \xi_t^{0, T_{N(t)}}(x) \leqslant L\right) \leqslant P(t - T_{N(t)} < j_n) + P\left(\sum_{x \in Z} \xi_t^{0, T_{N(t)}}(x) \leqslant L; t - T_{N(t)} > j_n\right)$$

We will now need the following coupling. Assume that A is a nonempty subset of Z; then we may construct ξ_t^0 and ξ_t^A in the same probability space in such a way that at all times t, ξ_t^A has more particles than ξ_t^0. For such a coupling see in Liggett[8] the proof of Theorem 1.9(c) in Chapter VI.

Observe now that if $t - T_{N(t)} > j_n$, then the last time the renewal process died out was at least j_n units time before t. By the preceding coupling if we start a renewal process at time $t - j_n$ with a single particle, it should have fewer particles than $\xi_t^{0, T_{N(t)}}$ at time t since the latter process had at least one particle at time $t - j_n$. Therefore by the Markov property

$$P\left(\sum_{x \in Z} \xi_t^{0, T_{N(t)}}(x) \leqslant L; t - T_{N(t)} > j_n\right) \leqslant P\left(\sum_{x \in Z} \xi_{j_n}^{0, T_{N(j_n)}}(x) \leqslant L\right)$$

So

$$P\left(\sum_{x \in Z} \xi_t^{0, T_{N(t)}}(x) \leqslant L\right) \leqslant P(t - T_{N(t)} < j_n) + P\left(\sum_{x \in Z} \xi_{j_n}^{0, T_{N(j_n)}}(x) \leqslant L\right) \tag{3.2}$$

Since the time between two renewals has infinite expectation, we have that $t - T_{N(t)}$ converges in probability to ∞ as t goes to infinity and so

$$\lim_{t \to \infty} P(t - T_{N(t)} < j_n) = 0$$

We first let t go to infinity, then n go to infinity, and use Lemma 2.1 in (3.2) to get (3.1).

Let A be a finite subset of Z and define the set of configurations

$$\mathscr{C} = \{\xi: \xi(x) = 1 \text{ for all } x \in A\}$$

Let $\bar{\xi}_0$ be any fixed initial configuration for the inhomogeneous contact process and recall that

$$\sigma = \inf\{t \geqslant 0: \bar{\xi}_t(0) = 1\}$$

We have that

$$P(\bar{\xi}_t \in \mathscr{C}) = P(\sigma < \infty; \bar{\xi}_t \in \mathscr{C}) + P(\sigma = \infty; \bar{\xi}_t \in \mathscr{C})$$

But if $\sigma = \infty$, then $\xi_t = \bar{\xi}_t$ for all $t \geqslant 0$ and since the critical contact process dies out, we have

$$\lim_{t \to \infty} P(\sigma = \infty; \bar{\xi}_t \in \mathscr{C}) = 0$$

For $\lambda > \lambda_c$ the above limit is also 0 and the reason this time is that if the supercritical contact process survives, it spreads linearly and therefore it must reach the origin eventually.

Consider now

$$0 \leqslant P(\sigma < \infty; \bar{\xi}_t^Z \in \mathscr{C}) - P(\sigma < \infty; \bar{\xi}_t \in \mathscr{C}) = P(\sigma < \infty; \bar{\xi}_t^Z \in \mathscr{C}; \bar{\xi}_t \notin \mathscr{C})$$

Define $\xi_t^{0, T_{N(t)}, \sigma}$ to be the renewal contact process starting at the random time σ with a single particle at the origin. Each time this process dies out after time σ we restart if with a particle at the origin. Observe that starting at σ, the three processes $\bar{\xi}_t$, $\bar{\xi}_t^Z$, and $\xi_t^{0, T_{N(t)}, \sigma}$ are goining to coincide on the interval $[l_t^\sigma, r_t^\sigma]$, where r_t^σ and l_t^σ are, respectively, the rightmost and leftmost particles of the renewal contact process $\xi_t^{0, T_{N(t)}, \sigma}$. Let $l(A)$ and $r(A)$ be, respectively, the two extremal points of A to be the left and the right. By the strong Markov property

$$\begin{aligned} &P(\sigma < \infty; \bar{\xi}_t^Z \in \mathscr{C}; \bar{\xi}_t \notin \mathscr{C}) \\ &\quad \leqslant \int_0^t P(\sigma \in ds)(P(r_t^s < r(A)) + P(l_t^s > l(A))) \\ &\quad + P(t < \sigma < \infty) \end{aligned} \tag{3.3}$$

By (3.1) and the symmetry of the contact process it is clear that

$$\lim_{t \to \infty} P(r_t^s < r(A)) = \lim_{t \to \infty} P(l_t^s > l(A)) = 0 \qquad \text{for all} \quad \lambda \geqslant \lambda_c$$

So by the dominated convergence theorem the integral in (3.3) converges to 0 as t goes to infinity and

$$\lim_{t \to \infty} (P(\sigma < \infty; \bar{\xi}_t^Z \in \mathscr{C}) - P(\sigma < \infty; \bar{\xi}_t \in \mathscr{C})) = 0$$

It is easy to see that the conditional distribution of $\bar{\xi}_t^Z$ conditional on $\{\sigma < \infty\}$ converges to the upper invariant measure $\bar{\mu}_\lambda$ (for a similar computation see Liggett,[8] p. 285). This completes the proof of Theorem 2. ∎

Note Added. Neal Madras (private communication) has an elementary proof of Corollary 1 which holds in any dimension.

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